Digital Systems: Exercises – Sheet 1

There are many exercises and you don't need to do all of them; you can select a few simple ones plus a few more difficult ones. Best is to solve them individually and compare results with those of another student. When the results are not equal, try to solve the exercise together with the other student and try to understand what and where the problem was. The professor can always help you. Success, Prof. Hans du Buf.

1.1 Convert from binary to decimal: (a) 1010111 (b) 0.00101 (c) 1011010.1010

1.2 Convert from octal to decimal: (a) 273 (b) 1021 (c) 16.423

1.3 Convert from hexadecimal to decimal: (a) 145 (b) A2C1 (c) 1A.B2

1.4 Convert from binary to hexadecimal and octal: (a) 10101110101101111011
(b) 011011110011110000000001

1.5 Convert from hexadecimal to binary: (a) 1A.B2 (b) A2C1 (c) F0AC29E

1.6 Convert from decimal to binary: (a) 122 (b) 98 (c) 48.45 (d) 195.98

1.7 Do the following additions and subtractions: (a) 101011 + 10111 (b) 1101 + 1110 + 1001 (c) 11101 – 10110 (d) 1100.010 – 1000.111

1.8 As 1.1, but with a run (corrida) of ones: 100111100

1.9 As 1.6, but without successive division: (a) 500 (b) 1000 (c) 2000

Sistemas Digitais: Exercises – Sheet 2

2.1 Convert from decimal (sign-magnitude) to C/10 using $10^n - N$, where n is the number of digits of the number N: (a) +123 (b) -48 (c) -323 (d) -2047
Note: you will need a sign digit!

2.2 Repeat 2.1, using C/10 = C/9 + 1

2.3 Convert from binary (sign-magnitude) to C/2 using $2^n - N$, where n is the number of digits of the number N: (a) 0001 0001 (b) 1011 0111 (c) 1000 0110 (d) 1000 1111

2.4 Repeat 2.3, using C/2 = C/1 + 1

2.5 Do the following subtractions in decimal (C/10): (a) 423 - 198 (b) 327 - 432

2.6 As 2.5, but in binary using C/2: (a) 79 - 42 (b) 87 - 99

2.7 Do the following operations in BCD: (a) 79 + 101 (b) 87 + 179 (c) 198 - 43 (d) 143 - 98

2.8 Convert the following Gray codes to binary and vice versa: (a) 1010 (b) 11011 (c) 11000010001

2.9 Do the following multiplications of the binary numbers in sign-magnitude in 4 and 8 bits: (a) 1110 x 1101 (b) 0001 0101 x 0000 1010

2.10 Convert the following numbers to binary in C/2 using 32 bits: (a) 512 (b) -1023 (c) -4 000 000

2.11 Convert to decimal the following binary numbers given in C/2:
(a) 1111 1111 1111 1111 1111 1111 0000 1100
(b) 1111 1111 1111 1111 1111 1111 1111 1111
(c) 0111 1111 1111 1111 1111 1111 1111 1111

Sistemas Digitais: Exercises – Sheet 3

3.1 Prove the following relations:
(a) $(\bar{a} + \bar{b})(a + b) = a \oplus b$
(b) $(ab + c)b = ab\bar{c} + \bar{a}bc + abc$
(c) $bc + ad = (b + a)(b + d)(a + c)(c + d)$
(d) $(ab + ac + bc)\overline{abc} = ab\bar{c} + ac\bar{b} + bc\bar{a}$

3.2 Simplify the following expressions:
(a) $F_1 = a\bar{c} + b\bar{c}d + a\bar{b}c + acd$
(b) $F_2 = b + \bar{a}\bar{b} + acd + a\bar{c}$
(c) $F_3 = b\bar{c}d + \bar{a}b\bar{d} + ab\bar{c} + a\bar{b}d + a\bar{c}d$

3.3 Find the complement ($\bar{F}$) of the following expressions (de Morgan):
(a) $F_1 = a + \bar{b}c$
(b) $F_2 = a(b + c) + b\bar{d}(\bar{a} + c)$

3.4 Implement the following functions with AND and OR gates plus invertors:
(a) $F_1 = \bar{a} + b(\bar{c} + \bar{d})$
(b) $F_2 = (\bar{a} + b)(\bar{b} + c)(ab + c)$

3.5 Construct (i) the truth table and (ii) the canonical forms SOP and POS of the following functions:
(a) $F_1 = a + b\bar{c}$
(b) $F_2 = ac + bc + ab$
(c) $F_3 = (a + \bar{b})(\bar{a} + \bar{b} + d)$

Sistemas Digitais: Exercises – Sheet 4

4.1 Find the minimum SOP functions of the following logical functions, assuming that variable a is the least significant one:
(a) $F(a, \ldots, d) = \sum(1, 3, 5, 7, 8, 10, 12)$
(b) $F(a, \ldots, c) = \sum(0, 1, 3, 4, 6, 7)$
(c) $F(a, \ldots, e) = \sum(0, 1, 3, 5, 6, 7, 8, 9, 10, 15, 16, 20, 21, 22, 23, 24, 25, 28, 29, 30, 31)$

4.2 Find the minimum POS functions of the following logical functions, assuming that variable a is the least significant one:
(a) $F(a, \ldots, c) = \sum(0, 1, 2, 5, 7)$
(b) $F(a, \ldots, d) = \sum(0, 1, 9, 10, 11)$
(c) $F(a, \ldots, e) = \sum(1, 2, 5, 6, 10, 11, 14, 15, 16, 17, 20, 21)$

4.3 Simplify the following functions with don't cares:
(a) $F(a, \ldots, c) = \sum(3, 5, \underline{0}, \underline{7})$
(b) $F(a, \ldots, d) = \sum(1, 2, 3, 5, 6, 7, 10, 11, \underline{9}, \underline{12}, \underline{15})$

4.4 Find the minimum SOP functions:
(a) $F(a, \ldots, d) = \prod(0, 4, 5, 6, 7, 8, 9, 10)$
(b) $F(a, \ldots, e) = \prod(1, 2, 5, 8, 9, 11, 12, 15, 17, 19, 21, 23, 24, 25, 28, 29, 30, 31, \underline{6}, \underline{7}, \underline{10}, \underline{14}, \underline{16}, \underline{18}, \underline{26}, \underline{27})$

4.5 A commission counts three members and each member can vote in favour (1) or against (0) proposals to be decided. What is the minimum SOP function which is 1 when at least two members (the majority) vote in favour? Implement the function with NAND gates.

4.6 A circuit has four inputs I_3, I_2, I_1, I_0 and two outputs O_1, O_0. The outputs indicate which input is 1. For example, if $I_3 = 1$, then $O_1, O_0 = 11$. If more than one input is 1, the outputs code the most significant input. Of all inputs, I_3 is the most significant and I_0 the least significant. At any time, at least one input must be 1. Implement the minimum circuit with NAND gates.

Sistemas Digitais: Exercises – Sheet 5

5.1 Design a circuit which converts a number of 3 bits in Gray code to the normal binary code, using XOR gates.

5.2 A multiplexer has has four inputs I_3, I_2, I_1, I_0 and one output O. Two "select" inputs S_1, S_0 determine which input will go to the output. Design this multiplexer with NAND gates.

5.3 A multiplexer with n select inputs can be used to realise any logical function with $n+1$ variables. Realise the function $f(a, b, c) = \sum(2, 3, 5, 6)$ with the multiplexer of problem 5.2.

5.4 A decoder with n inputs selects one of 2^n outputs. For example, one with 2 inputs selects one of four outputs. The selected output will be 1 whereas the other outputs will be 0. Realise a 1-of-4 decoder with NAND gates and invertors.

5.5 A comparator is a circuit which compares two binary numbers A and B. It has three outputs: $A < B$, $A > B$ and $A = B$. Construct a comparator (using the most convenient gates) when A and B have two bits.

5.6 An adder ("full adder") of 1 bit has three inputs: bit A_i of number A, bit B_i of number B, and the carry bit C_{i-1} of the neighbouring adder. It has two outputs: the sum S_i and the carry C_i. Realise the adder using the most convenient gates.

5.7 Design a high-speed adder with inputs A_1, A_0 and B_1, B_0 plus carry input C_{in}, using a SOP circuit.

Sistemas Digitais: Exercises – Sheet 6

6.1 Design a binary counter which counts input pulses modulo 4 (0, 1, 2, 3, 0, ...); (i) with flip-flops type SR, (ii) with FFs type JK.

6.2 A counter mod 8 with FFs type T and D.

6.3 A counter with 3 bits (mod 8) which counts in Gray code. Which type of FF (SR, T or D) yields the most simple counter, i.e., with the least number of gates?

6.4 An incremental counter mod 10 (0, 1, ... 9, 0, ...) with the type of FF you prefer, using don't cares.

6.5 As 6.4, but a decremental one (0, 9, ... 1, 0, ...).

6.6 A counter mod 16 chip has a clock input sensitive to positive flanks, an active-low reset input $\bar{R}$, and four outputs 8,4,2,1. Using such chips, design counters for 60 seconds and minutes, plus 24 hours.

Sistemas Digitais: Exercises – Sheet 7

7.1 There are two railroad tracks which join into one track. Hundred meters before the junction there are two sensors S1 and S2 which detect the presence of coming trains. Twenty meters before the junction there are two signals F1 and F2 (F=0: red light; F=1: green light). Only one train can pass, and when one train passes and another train approaches, the other one has to wait. Design the state diagram, make the state table and simplify the table. Construct the control circuit with FFs type D.

7.2 A circuit has one input and one output. The output is 1 ONLY when the circuit detects in 5 successive input bits three ones and two of these occur in the first three bits. Example: 011.01. If a started sequence of 5 bits cannot create an output 1, the circuit returns to the initial state and a new sequence starts. Analyse the problem and make the minimum state table.

7.3 A building has three floors and an elevator. Each elevator door has a button to call the elevator (C0, C1, C2). In the elevator there are three buttons to select the floor level (F0, F1, F2). Assume that only one button F may be pressed. After each movement, the elevator stays at the floor where it went. Analyse the problem by designing the state diagram and make the state table.

7.4 A door is protected with an electronic lock and a decimal keyboard. Only the code 3285 opens the lock. The control circuit returns to its initial state (i) after opening the lock and (ii) when a started sequence is or becomes wrong. If a person presses two or more keys simultaneously, a priority encoder integrated in the keyboard selects the highest key. Analyse the problem (state diagram etc.).

Sistemas Digitais: Exercises – Sheet 8 (extra)

8.1 Do the following multiplications in binary, (i) using one table, and (ii) step by step: (a) 10 x 10 (b) 7 x 14 (c) 27 x 54

8.2 Do the following divisions in binary: (a) 70 / 7 (b) 70 / 10 (c) 100 / 10 (d) 100 / 8 (e) 114 / 8

8.3 Introduce variable d in the POS form $(a + b)(a + c)$

8.4 Simplify $a(b + c) + bd + cd + be + ce + \bar{a}(d + e)$

8.5 Simplify $\overline{\overline{(\bar{a} + \bar{b})}} \cdot \overline{\overline{(\bar{c} + \bar{d})}}$

8.6 Show that $a \oplus \bar{b} = \bar{a} \oplus b = \overline{a \oplus b}$

8.7 Simplify (a) $a \oplus b \oplus 1$ and (b) $a \oplus b \oplus 0$

8.8 Simplify the following expressions (i) algebraically, (ii) with Karnaugh map, and (iii) return to (i) while looking at the Karnaugh map:
$F_1 = \bar{c}b + cba + cb\bar{a}$
$F_2 = b\bar{a} + cb + \bar{c}ba$
$F_3 = ca + \bar{b}\bar{a} + \bar{c}b\bar{a} + cb\bar{a}$
$F_4 = da + b\bar{a} + c\bar{b}\bar{a} + \bar{d}\bar{c}\bar{b}\bar{a} + d\bar{c}\bar{b}\bar{a}$
$F_5 = c\bar{b} + dc + \bar{d}cb$

SD – Exame de 12.1.2007

- Sem consulta e sem calculadora
- Tempo: 120 minutos

1. Faça uma tabela com as representações em sinal-magnitude, complemento para 1 e complemento para 2, em 12 bits, dos números decimais +999 e –999. Dos códigos complemento para 2, quais as representações octais e hexadecimais?

2. Faça a seguinte divisão em binário: 114/8.

3. Um μprocessador tem uma instrução **XOR F1,F2,D** (F=fonte; D=destino). Há dois números nos registos 5 e 6. O que aconteceu com os dois números depois da execução das instruções seguintes?

```
xor R5,R6,R6
xor R5,R6,R5
xor R5,R6,R6
```

Justifique a sua resposta!

4. Simplifique a função $f(x_4 \cdots x_0) = \sum(2, 4, 5, 13, 14, 18, 19, 22, 26, 27, 28, \underline{6}, \underline{10}, \underline{12}, \underline{20}, \underline{30})$ nas formas SOP e POS e desenhe o circuito mais simples (com portas NAND no caso SOP ou com portas NOR no caso POS). Nota: x_0 é a variável menos significativa.

5. Faça o desenho de um somador de duas vezes dois bits ($s_2 s_1 s_0 = a_1 a_0 + b_1 b_0$).

6. Um amigo interessado na música e sintetizadores quer experimentar com sinais diferentes. Ele gostaria criar a sequência binária seguinte: 0112334556770112... (os códigos impares têm dupla duração). Ajude o amigo e faça o desenho do contador, utilizando 3 ou 4 FFs tipo D. Pode assumir que o contador arranca sempre no estado 0.

Boa sorte!

SD – Recurso de 5.2.2007

- Sem consulta e sem calculadora
- Tempo: 120 minutos

1. Faça uma tabela com as representações em sinal-magnitude, complemento para 1 e complemento para 2, em 12 bits, dos números decimais +1234 e –1234. Dos códigos complemento para 2, quais as representações octais e hexadecimais?

2. Utilizando apenas a função XOR ($\oplus$), dê **pelo menos** três expressões da função $F(a,b,c) = \sum(0,3,5,6)$. A variável a é a mais significativa.

3. Simplifique algebricamente a função $F = G + \bar{a}bcd + \bar{a}\bar{b}c + \bar{a}bc\bar{d}$ aonde $G = a \oplus \bar{c}$.

4. Simplifique a função $f(x_4 \cdots x_0) = \sum(0,1,4,8,9,14,16,22,23,28,31,\underline{6},\underline{12},\underline{20},\underline{24},\underline{30})$ nas formas SOP e POS e desenhe o circuito mais simples (com portas NAND no caso SOP ou com portas NOR no caso POS). Nota: x_0 é a variável menos significativa.

5. Faça o desenho de um multiplicador de duas vezes dois bits ($p_3p_2p_1p_0 = a_1a_0 \times b_1b_0$).

6. Um amigo interessado na música e sintetizadores quer experimentar com sinais diferentes. Ele gostaria criar a sequência binária seguinte: 0112334556770112... (os códigos impares têm dupla duração). Ajude o amigo e faça o desenho do contador, utilizando 3 ou 4 FFs tipo D. Pode assumir que o contador arranca sempre no estado 0.

Boa sorte!