Digital Systems: Tips 'n Tricks

Also called Questions and Answers, or: the Flying Circus of Digital Systems...

1. Conversion decimal to binary

If you know the powers of two the conversion can be more simple. For example, convert 1000 in decimal to binary: 1000 = 1024 - 24. Since the subtraction can be messy because of borrowing, the easiest solution is to apply 1023 - 23:

```
    1024 512 256 128 64 32 16 8 4 2 1

    1 0 0 0 0 0 0 0 0 0 0  =  1024   ;   1 0 1 1 1  =  23

    0 1 1 1 1 1 1 1 1 1 1  =  1023
                1 0 1 1 1  =    23
  - ---------------------
      1 1 1 1 1 0 1 0 0 0
```

As you can see, this boils down to simply deleting corresponding ones!

2. Borrowing in subtractions

Above, in Tip 1, you have seen that subtracting one (1024 - 1 = 1023) results in "a run of ones." The same principle is used in subtractions, for example 8 - 3 = 5:

```
   1 0 0 0  =  8

         1  =  1
   0 1 1 1  =  7  (now there are two ones in the first position!)
       1 1  =  3
 - -------
     1 0 1  =  5
```

This rule can (must) be applied in numbers with N bits, processing the necessary sub-strings of ones.

3. Where does the name "two's complement" come from?

The name "two's complement" or C/2 comes from the conversion of fractions, namely by the subtraction of the positive fraction from 2. We can check this by first applying the normal rule (C/2 = C/1 plus 1)

```
   s . x x x x x  (s = sign; x = bit with weight 1/2 1/4 1/8 1/16 1/32 etc.)

   0 . 0 1 1 0 0  (0.375 = 0.25 + 0.125 ; positive so s = 0)

   1 . 1 0 0 1 1  (C/1)
                1  (plus 1)
 + -------------
   1 . 1 0 1 0 0  (negative so s = 1)
```

Now we can check what happens when we subtract the positive fraction from 2:

```
  1 0 . 0 0 0 0 0  (2)
    0 . 0 1 1 0 0  (the fraction)
- ---------------
    1 . 1 0 1 0 0
```

As you see, the result is the same. In order to check the result, we must convert the negative number to positive, using one of the two solutions explained above:

```
    C/2 = C/1 + 1                      C/2 = 2 - fraction

    1 . 1 0 1 0 0  (negative)          1 0 . 0 0 0 0 0  (2)
                                         1 . 1 0 1 0 0  (negative)
    0 . 0 1 0 1 1  (C/1)             - ---------------
                1  (plus one)            0 . 0 1 1 0 0
  + -------------
    0 . 0 1 1 0 0  (positive)
```

4. Can C/1 and C/2 only be applied to binary numbers?

No, it can be applied to numbers with any base (radix). However, in decimal, for example, C/1 becomes C/9 and C/2 becomes C/10. The C/9 was used in the Curta (www.curta.org), a mechanical calculator invented by the Jewish fine-mechanic Curt Herzstark, who managed to survive the NAZI concentration camp Buchenwald. In 1970 a Curta cost about 125 dollars. OK, let us see some examples, but only 53 - 26 and 26 - 53. First using C/9:

```
  53 - 26 = 53 + (-26) = 27  ;  C/9 of 26 is 73 or 973 (3 digits)

   53                              053
   73                              973
+ ---                           +  ----
  126  (carry!)                  1026  (carry!)

   26                              026
    1  (end-around carry)            1  (end-around carry)
+  --                           +   ---
   27                              027
```

The second case (to the right) is how it should be done, with a leading 0 digit in the case of positive and a leading 9 digit in the case of negative numbers. Now let us see 26 - 53 = -27:

```
  -53 = 46 or -053 = 946 in C/9

   26                              026
   46                              946
+ ---                           +  ---
   72 (gives 27 with C/9 BUT       972  (this is negative (9 sign)
       72 is not negative!)             and C/9 gives 027)
```

The same way we can take a look at C/10 = C/9 + 1:

```
 53 - 26                          26 - 53
-26 = 973 + 1 = 974              -53 = 946 + 1 = 947

 053                              026
 974                              947
+ ----                           + ---
 1027  (carry!)                   973  (negative and C/10 conversion
  027  (ignore carry)                    gives 026 + 1 = 027)
```

Now a question for the students: why was C/9 implemented in the Curta and not C/10 ?

To be continued?
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